Annals of Finance (2007) 3: 389–409
DOI 10.1007/s10436-006-0049-8

RESEARCH ARTICLE

Jean-Paul Décamps · Bertrand Djembissi

Switching to a poor business activity: optimal capital structure, agency costs and covenant rules

Received: 28 September 2005 / Accepted: 30 June 2006 / Published online: 1 August 2006

Abstract We address the issue of modeling and quantifying the asset substitution problem in a setting where equityholders decisions alter both the volatility and the return of the firm cash flows. Our results contrast with those obtained in models where the agency problem is reduced to a pure risk-shifting problem. We find larger agency costs and lower optimal leverages. We identify the bankruptcy trigger written in debt indenture, which maximizes ex-ante firm value, given that equityholders will ex-post be able to risk-shift. Our model highlights the tradeoff between ex-post inefficient behavior of equityholders and inefficient covenant restrictions.

Keywords Capital structure · Stockholder–debtholder conflict · Covenant rules

JEL Classification Numbers G30 · G32 · G33

1 Introduction

The asset substitution problem, first documented by Jensen and Meckling (1976), results from the incentives of equityholders to extract value from debtholders by avoiding safe positive net present value projects. This implies a decrease in the value of the firm, as a result of a decrease in the value of the debt and a smaller increase in the value of the equity. This opportunistic behavior of equityholders is incorporated into the price of debt and the ex-ante solution to this agency problem

J.-P. Décamps
GREMAQ-IDEI, Université de Toulouse 1, 21 Allée de Brienne, 31000 Toulouse, France

B. Djembissi
GREMAQ, Université de Toulouse 1, 21 Allée de Brienne, 31000 Toulouse, France

J.-P. Décamps (✉)
Europlace Institute of Finance, 39-41 rue Cambon, 75001 Paris, France
E-mail: decamps@cict.fr

is therefore to issue less debt. As a result, the optimal capital structure of the firm highlights the benefit of issuing debt because of tax benefits, and the cost of issuing debt because of both asset substitution problem and bankruptcy costs. It has long been recognized that such a standard equityholder–debtholder conflict might be a key for understanding observed behavior of firms. It is for instance well documented, see Graham (2000), that firms tend to choose large amount of equity in their capital structure and set debt levels well below what would maximize the tax benefits of debt.

Continuous time contingent-claims analysis offers a natural setting for modeling and quantifying the asset substitution effect. The prototype of this approach is the model of Leland (1998), in which equityholders, once debt is in place, can choose continuously between a high and a low volatility level for the firm's assets. Leland (1998) studies the impact of equityholders' ex-post flexibility to choose volatility on the firm's optimal capital structure and finds that agency costs restrict leverage and debt maturity and increase yield spreads. In his environment, agency costs of debt due to the asset substitution effect amount to about 1.5% of firm's value which is far less than the tax benefits of debt; bond covenants that restrict equityholders from adopting the high volatility are little value enhancing; more surprisingly the optimal leverage when there is an agency problem is larger than the optimal leverage of a firm that cannot increase risk. Leland (1998) applies his analysis to the study of risk management. Specifically, a low choice of volatility indicates an effective hedging strategy whereas a larger volatility indicates that the firm abandons its hedge and operates at a higher risk. It is suggested that derivatives can be used for implementing such a hedging strategy. Indeed, if derivatives are fairly priced and transaction costs are minimal, it is reasonable to consider that switching is reversible and that it occurs at no cost. Leland (1998) methodology is actually particularly attractive to study financial asset substitution and natural ingredients of his model are that equityholders choices only affect the volatility of the firm's assets and that the choice of the volatility is fully reversible.

In this paper, we propose a dynamic model of capital structure along the lines of Leland (1998). Our specificity is to adopt the view that the asset substitution effect can be also explained by bad investments rather than by simply pure excessive risk taking. According to Bliss (2001) this agency problem may be fundamental: "Poor (apparently irrational) investments are as problematic as excessively risky projects (with positive risk-adjusted returns)". In particular Bliss (2001) reviews several empirical articles that conclude that bank failures are often provoked by bad investments rather than bad luck (and excessive risk taking). This leads us to consider a model in which equityholders have the opportunity to change the firm strategy or its industrial project thereby altering both the risk adjusted expected growth rate and the volatility of the cash flows generated by the firm's assets. This change leads to difficult assets reallocation and can be undertaken only one time. This last consideration motivates our modeling assumption to consider as irreversible the equityholders switching decision. Doing this, our model is more concerned with *real asset* substitution rather than with *financial asset* substitution. We discuss further this point in Section 2.

Specifically, we consider that the firm's activity generates a *safe* lognormal cash flows process characterized by a given risk-adjusted expected growth rate and a given volatility. At any time equityholders have the opportunity to switch in an irreversible way to a *poor* activity. The adoption of the poor activity lowers the risk-adjusted expected growth rate of the cash flows process and increases its volatility. We therefore consider that two problems jointly define asset substitution, (i) a pure risk-shifting problem acting on the volatility of the growth rate of the cash flows, and (ii) a first order stochastic dominance problem acting on the risk-adjusted expected growth rate of the cash flows. We identify situations where equityholders adopt the poor activity and take a negative net present value decision. We analyze the loss in the firm value that is generated by this decision. We then investigate how covenant rules written in the debt indenture can protect debtholders and reduce the amount of these agency costs.

More precisely, in our model, debt is a coupon bond with infinite maturity and coupon payment offers tax deduction. As in Leland (1998), we consider endogenous bankruptcy. That is, equityholders have the option to decide when to cease paying the coupon and to declare bankruptcy. The bankruptcy policy is therefore chosen to maximize the value of equity, given the limited liability of equityholders and the debt structure. At each instant of time equityholders can switch in an irreversible way to the poor business activity. Switching generates agency costs whose magnitude is defined by the difference between the optimal firm value when the switching policy can be contracted ex-ante (before debt is in place) and the optimal firm value when the switching decision policy is taken ex-post (that is after debt is in place). In each case the optimal capital structure is characterized by the coupon rate that maximizes the initial firm value. The tradeoff underlying the model is as follows. On the one hand equityholders have incentives to switch to the poor activity because it increases their option value to declare bankruptcy. On the other hand switching strategy entails an opportunity cost since it lowers the (risk adjusted) instantaneous return of the cash flows. We show that a drop of the cash flows can throw equityholders in a "gambling for resurrection" situation which leads them to choose the poor business activity despite its lower risk adjusted expected return.

Our results contrast with the literature where the asset substitution problem is reduced to a pure risk-shifting problem. For example, depending on the severity of the agency problem, agency costs of debt at the optimal leverage can be large (more than 7% of the value of the firm). Accordingly, optimal leverage when an agency problem exists is lower than that of a firm that cannot change its activity. We pursue the analysis recognizing that covenants written in the debt indenture forcing equityholders to go bankrupt modify switching incentives and affect the level of agency costs. We identify the optimal covenant rule, that is, the bankruptcy trigger written in debt indenture, which maximizes ex-ante firm value, given that equityholders will ex-post be able to risk-shift. The solution takes the form of a "bang-bang" control. If the agency problem is severe enough then, the optimal covenant rule is defined as the lower bankruptcy trigger that fully eliminates any risk-shifting incentives. If on the contrary the agency problem is not severe enough then, any covenant rule worsens things and it is better to let equityholders to switch to the poor activity and to default strategically. Our model highlights the tradeoff between ex-post inefficient equityholders behavior and inefficient covenant restrictions.

Our paper contributes to a series of recent papers that have also considered the issue of risk-shifting in a contingent-claims analysis setting. For instance, Henessy and Tserlukevich (2004) focus on financial asset substitution and study the role of Warrant in solving agency problems in a setting with dynamic volatility choice.

They find that warrants mitigate asset substitution but exacerbate the agency problem of premature default. Ju and Ou-Yang (2005) show that, in a dynamic model in which the firm issues debt multiple times, the incentives of equityholders to increase the volatility of the firm's assets are reduced. Childs et al. (2005) provide a numerical model which accommodates both asset substitution and flexibility to increase or decrease the debt level at maturity dates. They find that financing flexibility encourages the use of short term debt and significantly reduces agency costs of investment distortions. Other related works on asset substitution are Mello and Parsons (1992), Mauer and Triantis (1994), Parrino and Weisbach (1999), Ericsson (2000), Décamps and Faure-Grimaud (2002), Mauer and Sarkar (2005).

The remainder of the paper is organized as follows: Section 2 presents the model and discusses our modeling assumptions, Section 3 analyzes optimal policies followed by equityholders, Section 4 defines and characterizes optimal capital structure and agency costs, Section 5 studies the role of covenants. Section 7 concludes. Proofs are in Appendix.

2 The model

Throughout the paper we denote by $W = (W_t)_{t\geq 0}$ a Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ and by $(\mathcal{F}_t)_{t\geq 0}$ the augmentation with respect to $\mathbb{Q}$ of the filtration generated by W. We denote by $\mathcal{T}$ the set of $\mathcal{F}_t$-adapted stopping times.

2.1 A simple model of the firm

We start by reviewing a standard model of a firm. The ideas and the results presented in this subsection are those of Leland (1994), Goldstein et al. (2001) or more recently Leland and Skarabot (2004). The underlying state variable X is the cash flows generated by the firm's activity (that is the firm's earnings before interest and taxes (EBIT)). We denote by "A" the activity in place and assume that the generated cash flows follow the stochastic differential equation

$$\frac{\mathrm{d}X_{t,A}}{X_{t,A}} = \mu_A \mathrm{d}t + \sigma_A \mathrm{d}W_t, \tag{1}$$

with initial condition $X_{0,A} = x$, where μ_A is the instantaneous risk-adjusted expected growth rate of the cash flows and σ_A the volatility of the growth rate. There is a risk free asset that yields a constant instantaneous rate of return $r > \mu_A$.[1] Agents are risk neutral and there are no informational asymmetries. The firm chooses its initial capital structure consisting of a perpetual coupon bond c that remains constant until equityholders endogenously default. In such a simple setting, the firm issues debt so as to take advantage of the tax shields offered for interest expenses. Failure to pay the coupon c triggers immediate liquidation of the firm. At liquidation, a fraction γ of the unlevered firm value is lost as a frictional cost. This leads to a liquidation value of the firm equal to

[1] We assume that the expected present value of the cash flows is positive and finite and therefore that $r > \mu_A$.

$$(1-\gamma)(1-\theta)\mathbb{E}\left[\int_0^{\infty} \mathrm{e}^{-rt} X^x_{t,A}\mathrm{d}t\right] = \frac{(1-\theta)(1-\gamma)x}{r-\mu_A}, \tag{2}$$

where θ is the tax rate on corporate income. Taking into account tax benefits and bankruptcy costs, the value of the levered firm is

$$v_A(x) = \mathbb{E}\left[\int_0^{\tau_L^A} \mathrm{e}^{-rt}\left((1-\theta)X^x_{t,A}+\theta c\right)\mathrm{d}t \;+\; \mathrm{e}^{-r\tau_L^A}\frac{(1-\theta)(1-\gamma)}{r-\mu_A}X^x_{\tau_L^A,A}\right],$$

where the stopping time τ_L^A defines the bankruptcy policy chosen by equityholders so as to maximize the value of their claim. Formally, the problem of the equityholders is: find the stopping time $\tau_L^A \in \mathcal{T}$ satisfying

$$\begin{aligned} E_A(x) &\equiv \sup_{\tau\in\mathcal{T}} \mathbb{E}\left[\int_0^{\tau} \mathrm{e}^{-rt}(1-\theta)\left(X^x_{t,A}-c\right)\mathrm{d}t\right] \\ &= \mathbb{E}\left[\int_0^{\tau_L^A} \mathrm{e}^{-rt}(1-\theta)\left(X^x_{t,A}-c\right)\mathrm{d}t\right]. \end{aligned} \tag{3}$$

Standard computations show that the optimal bankruptcy policy is a trigger policy defined by the stopping time $\tau_L^A = \inf\{t \geq 0 \text{ st } X_{t,A} = x_L^A\}$ with $x_L^A = -\frac{\alpha_A}{1-\alpha_A}\frac{c}{r}\frac{1}{\nu_A}$ where ν_A denotes the ratio $\frac{1}{r-\mu_A}$ and α_A denotes the negative root of the quadratic equation $y(y-1)\frac{\sigma_A^2}{2} + y\mu_A = r$. This implies the following expressions for the equity value $E_A(x)$, the firm value $v_A(x)$:

$$\begin{cases} E_A(x) = (1-\theta)\left\{x\nu_A - \frac{c}{r} + \left(\frac{c}{r} - x_L^A\nu_A\right)\left(\frac{x}{x_L^A}\right)^{\alpha_A}\right\} & \text{if } x > x_L^A, \\ E_A(x) = 0 & \text{if } x \leq x_L^A \end{cases} \tag{4}$$

and,

$$\begin{cases} v_A(x) = (1-\theta)x\nu_A + \frac{\theta c}{r} - \left(\frac{\theta c}{r} + x_L^A\gamma(1-\theta)\nu_A\right)\left(\frac{x}{x_L^A}\right)^{\alpha_A} & \text{if } x > x_L^A, \\ v_A(x) = (1-\gamma)(1-\theta)x\nu_A & \text{if } x \leq x_L^A. \end{cases}$$

The debt value $D_A(x)$ satisfies the relation

$$D_A(x) = v_A(x) - E_A(x) = \mathbb{E}\left[\int_0^{\tau_L^A} \mathrm{e}^{-rt}c\mathrm{d}t \;+\; \mathrm{e}^{-r\tau_L^A}\frac{(1-\theta)(1-\gamma)}{r-\mu_A}X^x_{\tau_L^A,A}\right]$$

or equivalently,

$$\begin{cases} D_A(x) = \frac{c}{r} - \left(\frac{c}{r} - x_L^A(1-\gamma)(1-\theta)\nu_A\right)\left(\frac{x}{x_L^A}\right)^{\alpha_A} & \text{if } x > x_L^A, \\ D_A(x) = (1-\gamma)(1-\theta)x\nu_A & \text{if } x \leq x_L^A. \end{cases}$$

The interpretation of (4) is standard. The equity value is equal to $(\nu_A x - \frac{c}{r})(1-\theta)$, the after tax net present value of equity if equityholders never declare bankruptcy, *plus* the option value associated to the irreversible closure decision at the trigger x_L^A. We denote in the sequel by $x_{PV}^A = \frac{1}{\nu_A}\frac{c}{r}$, the trigger that equalizes to zero the present value of equities under perpetual continuation. Note that, in line with the real option theory, the bankruptcy trigger x_L^A chosen by the equityholders is smaller than the net present value trigger x_{PV}^A.

As usual in such a classical setting, the optimal capital structure is then characterized by the coupon c to be issued that maximizes the initial firm value.

2.2 A simple model of the firm with risk flexibility

We now extend this standard model of capital structure by considering that, at any time, equityholders have the option to switch to a *poor* business activity (referred as "B" activity) that lowers the drift and increases the volatility of the cash flows. There is no monetary cost to change the activity but the decision to switch is assumed irreversible. Specifically, the poor activity "B" generates cash flows ("EBIT") satisfying the stochastic differential equation

$$\frac{\mathrm{d}X_{t,B}}{X_{t,B}} = \mu_B \mathrm{d}t + \sigma_B \mathrm{d}W_t, \tag{5}$$

with $\mu_B < \mu_A$ and $\sigma_B > \sigma_A$.

The key inequalities $\mu_A > \mu_B$ and $\sigma_B > \sigma_A$ characterize the tradeoff that drives our model. Because of limited liability equityholders will be tempted to choose the riskier activity (that is the largest possible volatility). However this choice has an opportunity cost since it induces a lower expected return ($\mu_B < \mu_A$). Intuitively, because of this opportunity cost, as long as the cash flows are large enough, changing the activity of the firm (that is switching to the poor activity) is not attractive and equityholders run the firm under the safe activity. However if the cash flows sharply drop, the lower expected return of the high risk activity may not dissuade equityholders from increasing the volatility of the cash flows. Saying it differently, the lower $\Delta\mu \equiv \mu_A - \mu_B$ with respect to $\Delta\sigma \equiv \sigma_B - \sigma_A$, the larger are the switching incentives of equityholders and therefore the more severe is the agency conflict. Accordingly, after switching, the liquidation value of the firm becomes

$$\frac{(1-\theta)(1-\gamma)x}{r-\mu_B}. \tag{6}$$

To sum up, in our model, equityholders have to decide (i) when to cease the activity in place and switch to the poor activity, (ii) when to liquidate. We refer to these two irreversible decisions as the switching/liquidation policy.

Some comments are in order before proceeding with the analysis of the model. The economic problem we address concerns *real asset* substitution. Suppose a firm confronted with the real option of changing its strategy or its industrial project. Such a change leads to a difficult assets reallocation that can be undertaken only one time. These one shot decisions may include reorganizing firms production or not, changing firm's activity or not, downsizing or not, going to a broader market or not. These particular real options decisions justify our assumption of irreversible switch. Irreversibility is nevertheless a limiting assumption and our model is less appropriate for studying *Financial asset* substitution related to speculation in derivatives (Henessy and Tserlukevich 2004) or hedging strategy by means of derivatives (Leland 1998). In our setting, where the expected value of the cash flows is also affected by equityhoders choices, an analogous interpretation to the Leland (1998) for hedging strategies would be to consider that a firm chooses activity "B" when it fails to invest sufficiently in risk management. This leads it to make poor investments in the sense that such investments does not present an attractive risk–return trade-off. Such an analysis would however require to relax the assumption of irreversible switch. Note that, with a reversible switch, it is not clear that the optimal switching strategy is defined by a single threshold, nor that switching for activity "B" is always socially suboptimal as it will be the case in our irreversible switching setting.

3 Optimal switching/liquidation policy

In order to study the optimal switching/liquidation policy, we first characterize situations where, whatever the initial value of the cash flows and the coupon c, (i) equityholders optimally decide to run the firm always under the safe activity, and (ii) equityholders immediately adopt the poor activity. We then study the more interesting case where always choosing the safe or the poor activity is not optimal.

In the previous section we derived $E_A(.)$, the equity value assuming equityholders run the firm under the safe activity (and optimally liquidate at time τ_L^A). In the same vein we can obtain $E_B(.)$, the equity value when equityholders run the firm always under the poor activity. We summarize this as follows.

Lemma 1 *Assume equityholders choose the poor Activity,* (*that is the dynamics of the cash flows obeys to the diffusion process* (5)) *then, the optimal liquidation policy is defined by the random time* τ_L^B *where* $\tau_L^B = \inf\{t \geq 0 \text{ st } X_{t,B} = x_L^B\}$ *with* $x_L^B = -\dfrac{\alpha_B}{1-\alpha_B}\dfrac{c}{r}\dfrac{1}{\nu_B}$. *In this case, the value of equity is defined by the equality*

$$E_B(x) = \mathbb{E}\left[\int_0^{\tau_L^B} \mathrm{e}^{-rt}(1-\theta)\left(X_{t,B}^x - c\right)\mathrm{d}t\right]$$

or equivalently,

$$\begin{cases} E_B(x) = (1-\theta)\left\{x\nu_B - \dfrac{c}{r} + \left(\dfrac{c}{r} - x_L^B\nu_B\right)\left(\dfrac{x}{x_L^B}\right)^{\alpha_B}\right\} & \text{if } x > x_L^B, \\ E_B(x) = 0 & \text{if } x \leq x_L^B \end{cases}$$

where ν_B *denotes the ratio* $\frac{1}{r-\mu_B}$ *and* α_B *denotes the negative root of the quadratic equation* $y(y-1)\frac{\sigma_B^2}{2} + y\mu_B = r$.

The two following lemma identify the cases where equity value $E(x)$ is either $E_B(x)$ (Lemma 2), or $E_A(x)$ (Lemma 3).

Lemma 2 *If $\mu_A = \mu_B$ and $\sigma_A < \sigma_B$ then, equityholders immediately choose the poor activity and liquidate the firm at the trigger x_L^B.*

Here, the switching decision is reduced to a pure risk shifting problem. Equity value is increasing and convex with respect to the cash flows x. In turn, this implies that equity value increases with the volatility of the cash flows. Formally, we have that for all $x \in (0, \infty)$, $E_A(x) \leq E_B(x)$ (see Figure 1). Consequently, equityholders immediately choose the poor activity, (that is the high risk activity), and liquidate at the trigger x_L^B. Note that the liquidation trigger is decreasing with the volatility and we have $x_L^B < x_L^A$. Since equityholders get nothing in the bankruptcy event, a necessary condition for never switching to the high-risk activity being always optimal is clearly $x_L^B > x_L^A$. The following lemma shows that it is also a sufficient condition.

Lemma 3 *If $x_L^A < x_L^B$ then, equityholders optimally never choose the poor activity and liquidate the firm at the trigger x_L^A.*

The condition $x_L^A < x_L^B$ ensures that $E_A(x) \geq E_B(x)$ for all values of x (see Figure 2). Equityholders cannot enjoy the high risk activity because the gain from increasing the volatility does not compensate the loss in the expected return.

In these two polar cases the tradeoff between increasing riskiness and decreasing expected return that drives our model is extreme. On the one hand, when increasing risk is costless (that is $\mu_A = \mu_B$) equityholders are better off choosing immediately the riskier activity and then never switch to the low risk activity. On the other hand, when $\Delta\mu$ is large with respect to $\Delta\sigma$, the high risk activity throws down bankruptcy and equityholders optimally always choose the low risk activity.

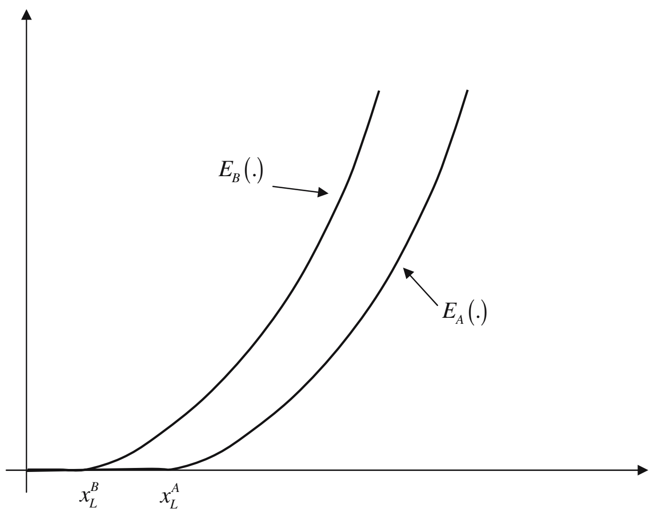

Fig. 1 $\mu_A = \mu_B$ and $\sigma_A < \sigma_B$

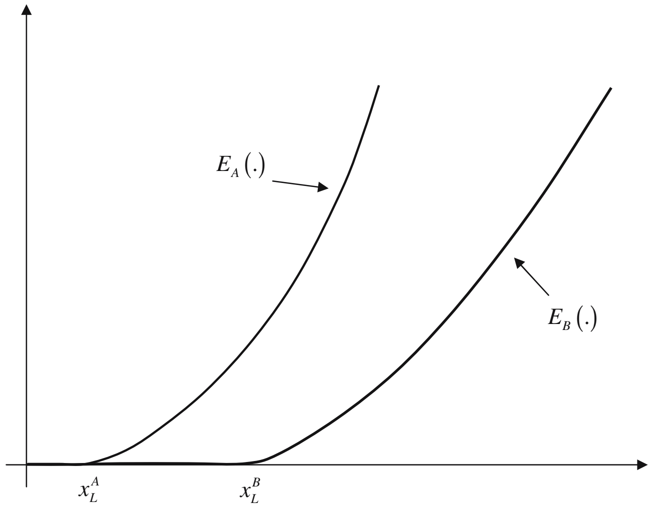

Fig. 2 $x_L^A < x_L^B, \mu_A > \mu_B, \sigma_A < \sigma_B$

We now study the more interesting case where neither choosing forever the poor activity or the safe activity is optimal. According to the two previous lemma, a necessary and sufficient condition for that is $x_L^B < x_L^A$ and $\mu_A > \mu_B$. Intuitively, switching to the poor activity is optimal for low values of the cash flows (since for $x_L^B < x < x_L^A$ we have $E_B(x) > 0$ and $E_A(x) = 0$), whereas for sufficiently large values of the cash flows it may be optimal to postpone the switching decision in order to benefit from the larger expected return of the safe activity.

Assuming equityholders start running the firm under the safe activity, their problem is to decide when to switch to the poor activity. Formally, equityholders solve the optimal stopping time problem: Find the stopping times $\tau_S^\star < \tau_L^\star \in \mathcal{T}$ satisfying

$$\begin{aligned}
E(x) \equiv (1-\theta) \sup_{\tau_S \in \mathcal{T}, \tau_L \in \mathcal{T}} & \left\{ \mathbb{E}\left[\int_0^{\tau_S} \mathrm{e}^{-rt} \left(X_{t,A}^x - c \right) \mathrm{d}t \right.\right. \\
& \left.\left. + \mathbb{E}\left[\int_{\tau_S}^{\tau_L} \mathrm{e}^{-rt} \left(X_{t,B}^{\tau_S, X_{\tau_S,A}^x} - c \right) \mathrm{d}t | \mathcal{F}_{\tau_S} \right]\right]\right\} \\
= (1-\theta) & \left\{ \mathbb{E}\left[\int_0^{\tau_S^\star} \mathrm{e}^{-rt} \left(X_{t,A}^x - c \right) \mathrm{d}t \right.\right. \\
& \left.\left. + \mathbb{E}\left[\int_{\tau_S^\star}^{\tau_L^\star} \mathrm{e}^{-rt} \left(X_{t,B}^{\tau_S^\star, X_{\tau_S^\star,A}^x} - c \right) \mathrm{d}t | \mathcal{F}_{\tau_S^\star} \right]\right]\right\}
\end{aligned} \tag{7}$$

where $X_{t,B}^{\tau_S, X^x_{\tau_S,A}}$ denotes the process $X_{t,B}$ that takes value $X^x_{\tau_S,A}$ at time τ_S. We show the following:

Proposition 1 *If $x_L^B < x_L^A$ and $\mu_A > \mu_B$ then, equityholders strategically switch to the poor activity at the random time $\tau_S^\star = \inf\{t \geq 0 \ \text{ st } \ X_t = x_S^\star\}$ and declare bankruptcy at the random time $\tau_L^B = \inf\{t \geq 0 \ \ s.t \ \ X_t = x_L^B\}$. The triggers $x_S^\star$ and x_L^B are defined by the relations*

$$x_S^\star = \left(\frac{(\alpha_B - \alpha_A)\nu_B}{(\nu_A - \nu_B)(1-\alpha_A)(-\alpha_B)}\right)^{\frac{1}{1-\alpha_B}} x_L^B, \quad \text{and} \quad x_L^B = -\frac{\alpha_B}{1-\alpha_B}\frac{c}{r}\frac{1}{\nu_B}.$$

The value of equity is defined by the equalities

$$\begin{cases} E(x) = (1-\theta)\Big\{x\nu_A - \dfrac{c}{r} - x_S^\star(\nu_A - \nu_B)\left(\dfrac{x}{x_S^\star}\right)^{\alpha_A} \\ \qquad + \left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_S^\star}\right)^{\alpha_A}\left(\frac{x_S^\star}{x_L^B}\right)^{\alpha_B}\Big\} \ \ \textit{if } x > x_S^\star, \\ E(x) = (1-\theta)\left\{x\nu_B - \frac{c}{r} + \left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B}\right\} \ \ \textit{if } x_L^B < x \leq x_S^\star, \\ E(x) = 0 \ \ \textit{if } x < x_L^B. \end{cases} \tag{8}$$

Our proposition deserves some comments. First, it shows that the conditions $x_L^B < x_L^A$ and $\mu_A > \mu_B$ are necessary and sufficient for switching from the safe activity to the poor activity being optimal. Second, it shows that the optimal switching policy is characterized by a switching trigger $x_S^\star > x_L^A$ that we derive explicitly. Figure 3 illustrates our proposition. Once the cash flows go below the switching trigger $x_S^\star$ equityholders optimally switch to the poor activity. Because this choice is by assumption irreversible, the equity value is then equal to E_B, the equity value under the poor activity. As long as the cash flows are larger than $x_S^\star$, the value of the option to switch is strictly positive and $E(x) > E_G(x)$.

In our setting, an approximate measure for the severity of the agency problem is the length of the interval $[x_L^B, x_L^A]$. Indeed the larger $\Delta\sigma$, the larger will be the length of the interval $[x_L^B, x_L^A]$, and the larger the switching trigger $x_S^\star$. On the contrary the larger $\Delta\mu$, the lower will be the distance between x_L^B and x_L^A. Ultimately, when $\Delta\mu$ is too large with respect to $\Delta\sigma$, the trigger x_L^B becomes larger than the trigger x_L^A, any incentive to choose the poor activity disappears and, according to Lemma 3, equityholders always choose the safe activity. It is interesting to compare the switching trigger $x_S^\star$ to the triggers $x_{PV}^A = \frac{c}{r}\frac{1}{\nu_A}$ and $x_{PV}^B = \frac{c}{r}\frac{1}{\nu_B}$ that equalize to 0 the net present value of equity under perpetual continuation when the firm is run, respectively with the safe activity and with the poor activity. In particular, when $x_{PV}^A < x_S^\star < x_{PV}^B$ the present value of equity evaluated at the switching point $x_S^\star$ is positive under the safe activity but negative under the poor activity. Equityholders nevertheless strategically switch to the negative net present value project at trigger $x_S^\star$ because the increase in their option value to declare bankruptcy compensates the loss in the net present value defined by the difference $\nu_A - \nu_B$.

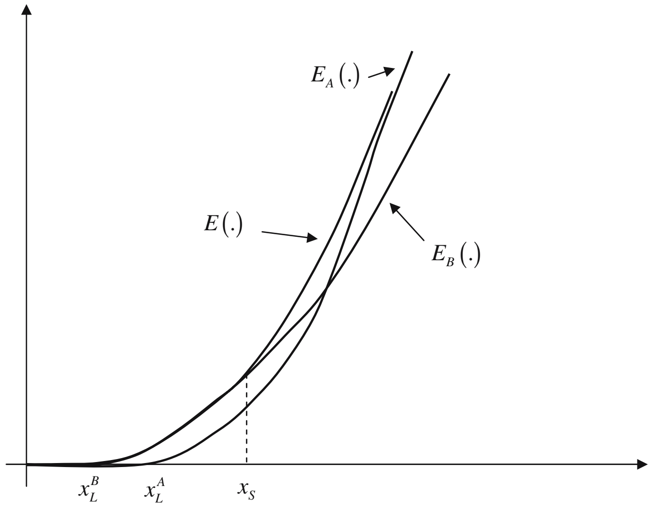

Fig. 3 $x_L^A > x_L^B, \mu_A > \mu_B, \sigma_A < \sigma_B$

We now give the ex-post firm value $v(x)$, that is the value of the firm when equityholders strategically switch at the trigger $x_S^\star$. We have

$$v(x) = \mathbb{E}\left[\int_0^{\tau_S} \mathrm{e}^{-rt}\big((1-\theta)X_{t,A}^x + \theta c\big)\,\mathrm{d}t + \mathrm{e}^{-r\tau_S} v_B\big(X_{\tau_S,A}^x\big)\right],$$

where

$$v_B(x) = \mathbb{E}\left[\int_0^{\tau_L^B} \mathrm{e}^{-rt}\big((1-\theta)X_{t,B}^x + \theta c\big)\,\mathrm{d}t + \mathrm{e}^{-r\tau_L^B}(1-\gamma)(1-\theta)\nu_B X_{\tau_L^B,B}^x\right].$$

Direct computations yield to

$$\begin{cases} v(x) = (1-\theta)x\nu_A + \frac{\theta c}{r} \\ \qquad -(1-\theta)x_S^\star(\nu_A - \nu_B)\left(\frac{x}{x_S^\star}\right)^{\alpha_A} \\ \qquad -\left(\frac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\left(\frac{x}{x_S^\star}\right)^{\alpha_A}\left(\frac{x_S^\star}{x_L^B}\right)^{\alpha_B} & \text{if } x > x_S^\star, \\ v(x) = (1-\theta)x\nu_B + \frac{\theta c}{r} \\ \qquad -\left(\frac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B} & \text{if } x_L^B < x \le x_S^\star, \\ v(x) = (1-\gamma)(1-\theta)x\nu_B & \text{if } x \le x_L^B \end{cases} \tag{9}$$

Let us comment briefly equation (9). For $x \le x_L^B$ the firm is all-equity financed, run by the former debtholders and we have $v(x) = (1-\gamma)(1-\theta)\mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}X_{t,B}^x\mathrm{d}t\right] = (1-\gamma)(1-\theta)x\nu_B$. For $x_L^B < x < x_S^\star$, the firm value is equal to the after tax present value of the cash flows when it is run under the poor activity $((1-\theta)x\nu_B)$ plus the present value of tax benefits $\left(\frac{\theta c}{r}\right)$ minus the discounted expected loss in case of bankruptcy $\left(\left(\theta c/r + x_L^B \gamma(1-\theta)\nu_B\right)\left(x/x_L^B\right)^{\alpha_B}\right)$. The amount of this loss is equal, at the bankruptcy trigger, to the loss of the tax benefits $\left(\frac{\theta c}{r}\right)$ plus the loss due to the bankruptcy cost $\left(x_L^B \gamma(1-\theta)\nu_B\right)$. For $x > x_S^\star$, the additional term $x_S^\star(1-\theta)\,(\nu_A - \nu_B)\left(\frac{x}{x_S^\star}\right)^{\alpha_A}$ represents the discounted expected loss in net present value that occurs at the switching trigger $x_S^\star$.

4 Optimal capital structure and agency costs

Equityholders' option to change the activity at the trigger $x_S^\star$ entails loss in value for debtholders and for the whole firm. If equityholders were able to commit to a certain management policy before debt is issued, this problem will disappear. Staying in the tradition of Leland (1998) we define agency costs as the difference between the optimal firm value when the switching policy can be contracted ex-ante (before debt is in place) and the optimal firm value when the switching decision policy is taken ex-post (that is after debt is in place). In each case the optimal capital structure is characterized by the coupon rate that maximizes the initial firm value. We now turn to the numerical implementation of our model and we analyze in this section, through several examples, some properties of the optimal capital structure and the magnitude of the agency costs. Table 1 lists the baseline parameters that support our analysis. Table 2 reports for different values of the couples (μ_A, σ_A) and (μ_B, σ_B) the optimal capital structure for the ex-ante case and for the ex-post case. The following observations can be made.

1. When the firm's activity policy can be committed ex-ante to maximize firm value, equityholders will never switch to the high risk activity. The optimal ex-ante firm value coincides in our setting with the optimal firm value when there is no risk flexibility. The agency costs, that can be very large, are highly sensitive to a change in $\Delta\mu$, the opportunity cost of choosing the high risk activity. Table 2 illustrates this point with agency costs dropping from 13.24 to 1.92% for a 2.5% increase of $\Delta\mu$. Accordingly, agency costs increase with $\Delta\sigma$ (that is agency costs increase when equityholders have more incentive to choose the high risk activity). In Table 2 agency costs increase from 1.02 to 13.24% when $\Delta\sigma$ goes from 5 to 30%.
2. The model predicts that the larger is the severity of the agency problem, the lower the optimal leverage ratios. Precisely, optimal leverages in presence of agency costs decrease relative to the ex-ante case where there is no risk flexibil-

Table 1 Base-case parameter values

γ	θ	r	x
0.4	0.35	0.06	5

Parameters for the base case: γ is the bankruptcy cost, θ the tax rate, r the fixed market interest rate and x the normalized initial cash flows value. The values that we consider in our analysis are standard in the continuous time corporate finance literature

Table 2 Optimal capital structure and magnitude of the agency costs for the ex-ante case and for the ex-post case, for different values of the couples (μ_A, σ_A) and (μ_B, σ_B)

	$v(x)$	$c^o(x)$	$L(\%)$	Y_S(bp)	AC(%)
$\sigma_A = 0.15$	$\Delta\sigma = 5\%$	$\mu_A = 0.015$		$\Delta\mu = 0.5\%$	
Ex-ante	92.05	4.77	74.61	95	–
Ex-post	91.11	4.54	72.07	91	1.02
$\sigma_A = 0.1$	$\Delta\sigma = 30\%$	$\mu_A = 0.015$		$\Delta\mu = 0.5\%$	
Ex-ante	96.34	5.03	80.51	49	–
Ex-post	83.58	2.37	43.84	47	13.24
$\sigma_A = 0.1$	$\Delta\sigma = 30\%$	$\mu_A = 0.03$		$\Delta\mu = 3\%$	
Ex-ante	148.37	7.88	83.91	33	–
Ex-post	145.52	7.32	79.62	32	1.92
$\sigma_A = 0.2$	$\Delta\sigma = 20\%$	$\mu_A = 0.03$		$\Delta\mu = 3\%$	
Ex-ante	135.88	7.08	72.54	118	–
Ex-post	132.98	6.34	67.1	111	2.13

$v(x)$ is the optimal firm value; $c^0(x)$ is the optimal coupon; L (in percentage of the firm value) is the optimal leverage (D/v) where the debt value D is equal to $v - E$; Y_S (in basis points) is the yield spread $(c/D - r)$ over the debt; AC (in percentage of the ex-ante firm value) is the magnitude of the agency costs

ity. In Table 2, leverage can drop by more than 35% with respect to the ex-ante case where there is no risk flexibility.

3. In our model, agency costs have no significant effect on yield spreads. The reason is that we focus on a pure switching problem between two activities. In particular, we do not consider an additional financing need at the switching trigger nor production costs for generating the cash flows. Remark however that yield spreads are lower in the ex-post case than in the ex-ante case. This result can be explained noting that optimal leverage in the ex-ante case is larger than optimal leverage in the ex-post case.

5 Covenants

Following Leland (1998) and many others, we have considered the case of endogenous bankruptcy (equityholders decide the time to go bankrupt). It is however also well documented that covenants written in the debt indenture can trigger bankruptcy. For instance, the so-called "cash flows based" covenant rule triggers bankruptcy as soon as the instantaneous cash flows X_t are not sufficient to cover coupon payments c to debtholders. This is the line followed by Kim et al. (1993), Anderson and Sundaresan (1996), Fan and Sundaresan (2000) or Ericsson (2000). The purpose of this section is to study how such covenant rules impact on the magnitude of agency costs, a task that seems to have been neglected in the literature.[2] In this section, we identify the optimal covenant rule, that is the bankruptcy trigger

[2] Ericsson (2000) is perhaps the only paper that addresses the issue of the magnitude of the asset substitution problem in a setting where bankruptcy is triggered by a covenant rule.

written in debt indenture which maximizes ex-ante firm value, given that equityholders will ex-post be able to risk-shift. Interestingly, the solution takes the form of a "bang-bang" control: If the agency problem is severe enough then, the optimal covenant rule is defined by the lower bankruptcy trigger that fully eliminates any risk-shifting incentives; if the agency problem is not severe enough then, it is better to let equityholders switch at the trigger $x_S^\star$ and then liquidate at the threshold x_L^B. In this latter case, the optimal covenant liquidation trigger coincides with the endogenous bankruptcy trigger chosen by equityholders. In the following section 5.1, we motivate and introduce the "no-switching based" covenant rule defined as the lowest liquidation trigger that eliminates any risk shifting incentives, and in Section 5.2, we turn to the study of the optimal covenant rule.

5.1 The "no switching based" covenant rule

We start by a simple remark. Under the so called "cash flows based" covenant rule, the exogenous liquidation threshold is $x_L = c$ and equity value $E_A(x)$ becomes

$$\begin{cases} E_A(x) = (1-\theta)\left\{x\nu_A - \dfrac{c}{r} + \left(\dfrac{c}{r} - c\nu_A\right)\left(\dfrac{x}{c}\right)^{\alpha_A}\right\} & \text{if } x > c, \\ E_A(x) = 0 & \text{if } x \le c. \end{cases} \tag{10}$$

Now, if one considers a non-declining industry (that is a positive instantaneous growth rate, $\mu_A > 0$) then, equity value (10) is concave in the current cash flows x, increasing in the rate of return μ and decreasing in the volatility σ.[3] Thus, equityholders are never tempted by the poor activity and the firm is liquidated at the exogenous trigger $x_L^{CF} = c$. Unfortunately, the fact that equityholders never switch to the poor activity does not imply that agency costs of debt are reduced. Quite on the contrary, numerical results show that rather than triggering premature bankruptcy at the threshold x_L^{CF}, it is socially optimal to let equityholders switch to the poor activity and liquidate at the threshold x_L^B lower than x_L^{CF}. This suggests that less strong covenants that restrict the firm from adopting the poor activity may be useful to reduce agency costs. This leads us now to introduce the "no-switching based" covenant rule defined as the lowest liquidation trigger such that the unique optimal policy for equityholders is never to switch to the poor activity.

Proposition 2 *The smallest liquidation trigger such that the switching problem disappears is given by*

$$x_L^{NS} = \frac{c}{r}\frac{\alpha_B - \alpha_A}{\nu_A(1-\alpha_A) - \nu_B(1-\alpha_B)}.$$

Some comments are in order. First, note that $x_L^{NS} < x_L^{CF}$. That is, "cash flows based" covenant rule is not necessary to give equityholders the incentives never

[3] This point is remarked by Leland (1994) who notes that, when debt is protected by a positive net worth covenant, equityholders will not gain by increasing firm risk and concludes that, in presence of potential agency conflict, protected debt may be the preferred form of financing despite having lower potential tax benefits. Leland (1994) does not however study the magnitude of agency costs at the optimal leverages nor remarks that positive net worth covenant can trigger inefficient premature bankruptcy.

to switch to the poor activity. Triggering bankruptcy at the lower trigger x_L^{NS} is sufficient. Second, the trigger x_L^{NS} is decreasing with the opportunity costs of switching ($\Delta\mu$). That is, when the difference in net present value of the two available activities increases, equityholders have less incentives to switch to the poor activity, and consequently, there is less need to engage in costly covenant restrictions to make them never choose the poor activity. Last, the following relation holds: $x_L^{NS} \geq x_L^A \Leftrightarrow x_L^A \geq x_L^B$. In words, the liquidation trigger x_L^{NS} is larger than x_L^A, (the optimal liquidation trigger when there is no switching) if and only if equityholders have indeed incentives to switch. This last remark gives us natural bounds for determining x_L^{SB}, the covenant rule (or second best) defined as the bankruptcy threshold which maximizes ex-ante firm value, given that equityholders will ex-post be able to risk-shift. Indeed, assume equityholders have incentives to switch ($x_L^A \geq x_L^B$), and consider a liquidation cash flows covenant at x_L. Taking $x_L \geq x_L^{NS}$ is clearly inefficient since it imposes liquidation whereas the firm would have continue under technology "A". On the other hand, equityholders never agree to continue firm's activity below the trigger x_L^B. It thus follows that the optimal covenant trigger x_L^{SB} lies in the interval $\left[x_L^B, x_L^{NS}\right]$.

5.2 Optimal covenant rule

Our result is based on the next proposition which derives the switching trigger $x_S(x_L)$ chosen by equityholders when bankruptcy is triggered by a cash flows covenant at $x_L \in \left[x_L^B, x_L^{NS}\right]$.

Proposition 3 *Consider a cash flows covenant at $x_L \in \left[x_L^B, x_L^{NS}\right]$, then equityholders react choosing a shifting trigger $x_S(x_L)$ given by*

$$x_S(x_L) = \left(\frac{\left(\frac{c}{r x_L} - \nu_B\right)(\alpha_B - \alpha_A)}{(\nu_G - \nu_B)(1 - \alpha_A)} \right)^{\frac{1}{1-\alpha_B}} x_L.$$

Furthermore, the mapping $x_L \mapsto x_S(x_L)$ is decreasing and satisfies the equalities $x_S\left(x_L^B\right) = x_S^\star$ and $x_S\left(x_L^{NS}\right) = x_L^{NS}$ where $x_S^\star$ is the risk shifting trigger associated to endogenous default.

The proof is easy and consists to replace in equation (8), $x_S^\star$ and x_L^B by $x_S(x_L)$ and x_L, and to maximize with respect to $x_S(x_L)$. The equality $x_S^\star\left(x_L^{NS}\right) = x_L^{NS}$ clearly corroborates the fact that x_L^{NS} is the lowest liquidation trigger which eliminates any risk-shifting incentives. Consider now an initial cash flows value x larger than x_L^{NS}, it then follows from Proposition 5.3 that the optimal covenant rule x_L^{SB} is defined by

$$x_L^{SB} = \arg \max_{x_L \in \left[x_L^B, x_L^{NS}\right]} v(x; x_L), \tag{11}$$

where, according to equation (9) and Proposition 3, the firm value $v(x; x_L)$ satisfies for $x \geq x_L^{NS}$ the relation

$$v(x;x_L) = (1-\theta)x\nu_A + \frac{\theta c}{r} - (1-\theta)x_S(x_L)\,(\nu_A-\nu_B)\left(\frac{x}{x_S(x_L)}\right)^{\alpha_A} \\ -\left(\frac{\theta c}{r} + x_L\gamma(1-\theta)\nu_B\right)\left(\frac{x}{x_S(x_L)}\right)^{\alpha_A}\left(\frac{x_S(x_L)}{x_L}\right)^{\alpha_B}. \tag{12}$$

It is easy to see from the first order condition that x_L^{SB} does not depend on the current value x of the cash flows. We have turned to numerical computations for solving the optimization problem (11). It follows from our study that the function $x_L \mapsto v(x;x_L)$ is U-shaped. This leads therefore to an optimal liquidation policy that consists in a binary choice. That is, $x_L^{SB} = x_L^B$ if $v\left(x;x_L^B\right) \geq v\left(x;x_L^{NS}\right)$ or, $x_L^{SB} = x_L^{NS}$ if $v\left(x;x_L^B\right) \leq v\left(x;x_L^{NS}\right)$. Furthermore, our numerical computations show that the inequality $v\left(x;x_L^B\right) \leq v\left(x;x_L^{NS}\right)$ is satisfied[4] for $\Delta\mu$ small with respect to $\Delta\sigma$ (that is when the agency problem is severe), whereas the reverse inequality is satisfied for $\Delta\mu$ large with respect to $\Delta\sigma$, (that is when the agency problem is not severe). To summarize, depending on the severity of the agency conflict, either it is optimal to let equityholders switch at $x_S^\star$ and liquidate at x_L^B; either it is optimal to close the firm at the lowest trigger that prevent equityholders from switching. This latter rule is the optimal one when the switching incentives are large (that is $\Delta\mu$ small with respect to $\Delta\sigma$). Covenant restrictions may be thus useful only provided that they fully deter the switching problem. Deterring risk shifting incentives is however costly for the firm and must be used only when the agency problem is severe enough. This result highlights the tradeoff between ex-post inefficient equityholders behavior and inefficient covenant restrictions.

Based on this analysis, we have numerically compared the firm values, or equivalently agency costs, when the liquidation policy is defined by the threshold[5] $x_L = x_L^{NS}$ and when liquidation is triggered by $x_L = x_L^B$. We have considered several environments as illustrated by Table 3. As announced, if the agency problem is not severe enough, covenants worsens things and it is optimal to let equityholders default strategically. However, if the risk shifting problem is severe, covenant at x_L^{NS} is value improving. For instance, in Table 3 when the agency problem is not severe enough, agency costs increase from 1.02% for the endogenous bankruptcy rule to 2.59% for the "no-switching based" covenant rule. On the contrary, when the agency problem is severe, convenant at x_L^{NS} reduces agency costs by more than 9% with respect to endogenous bankruptcy (accordingly, optimal leverage increases from 43.84 to 71.30%). In the two last environments presented in Table 3, the threshold x_L^{NS} is very close to the threshold x_L^A and the agency costs are negligible.

[4] Explicit computations show that, for a fixed current cash flow $x \geq x_L^{NS}$ and a fixed coupon c, the inequality $v(x;x_L^B) \leq v(x;x_L^{NS})$ only depends on the deep parameters of the model, that is $\mu_A, \mu_B, \sigma_A, \sigma_B, \theta, \gamma$.

[5] Under the "no-switching based" covenant rule, the ex-post value of the firm is given by the following expression:

$$\begin{cases} v(x) = (1-\theta)x\nu_A + \dfrac{\theta c}{r} - \left(\dfrac{\theta c}{r} + (1-\theta)x_L^{NS}\gamma\nu_A\right)\left(\dfrac{x}{x_L^{NS}}\right)^{\alpha_A} & \text{if } x > x_L^{NS}, \\ v(x) = (1-\gamma)(1-\theta)x\nu_A \ \text{ if } \ x \leq x_L^{NS}. & \end{cases}$$

Table 3 Optimal capital structure and magnitude of the agency costs when bankruptcy is endogenous and when bankruptcy is triggered by the "no-switching based" covenant rule

	$v(x)$	$c^{\mathrm{o}}(x)$	$L(\%)$	Y_S(bp)	AC(%)
$\sigma_A = 0.15$	$\Delta\sigma = 5\%$	$\mu_A = 0.015$		$\Delta\mu = 0.5\%$	
Ex-post case with endogenous bankruptcy	91.11	4.54	72.07	91	1.02
Ex-post case with no switching based covenant	89.64	4.19	73.22	38	2.59
$\sigma_A = 0.1$	$\Delta\sigma = 30\%$	$\mu_A = 0.015$		$\Delta\mu = 0.5\%$	
Ex-post case with endogenous bankruptcy	83.58	2.37	43.84	47	13.24
Ex-post case with no switching based covenant	92.50	4.23	71.30	41	3.99
$\sigma_A = 0.1$	$\Delta\sigma = 30\%$	$\mu_A = 0.03$		$\Delta\mu = 3\%$	
Ex-post case with endogenous bankruptcy	145.52	7.32	79.62	32	1.92
Ex-post case with no switching based covenant	147.33	7.61	82.41	27	0.70
$\sigma_A = 0.2$	$\Delta\sigma = 20\%$	$\mu_A = 0.03$		$\Delta\mu = 3\%$	
Ex-post case with endogenous bankruptcy	132.98	6.34	67.1	111	2.13
Ex-post case with no switching based covenant	135.88	7.08	72.54	118	0

$v(x)$ is the optimal firm value; $c^0(x)$ is the optimal coupon; L (in percentage of the firm value) is the optimal leverage (D/v) where the debt value D is equal to $v - E$; Y_S (in basis points) is the yield spread $(c/D - r)$ over the debt; AC (in percentage of the ex-ante firm value) is the magnitude of the agency costs

6 Conclusion

Most of the literature on asset substitution in a contingent-claims analysis setting considers the case in which the growth rate of the cash flows remains constant while the volatility of the cash flows increases by moral hazard. In this paper we adopt the view that the level of agency costs can be due also to bad investments rather than by simply pure excessive risk taking. This leads us to consider a model in which both the drift and the volatility of the cash flows are altered by equityholders decisions. We characterize explicitly equityholders optimal strategies and show using a numerical implementation of the model, that the risk of switching to a poor business activity drastically decreases firm value and optimal leverages. We furthermore investigate the role of positive net worth covenant written on the debt in reducing or exacerbating the magnitude of agency costs. We find that the optimal covenant rule takes the form of a "bang-bang" control. If the agency problem is severe enough, it is optimal to force bankruptcy at the lowest trigger that prevents equityholders from switching to the poor activity. If, on the contrary, the agency problem is not severe then, it is better to let equityholders switch and liquidate at their convenience.

Appendix

Proof of Lemma 2 Let $\nu = \frac{1}{r-\mu}$, α_σ denote the negative root of the quadratic equation $\frac{1}{2}\sigma y^2 + \left(\mu - \frac{1}{2}\sigma^2\right) y - r = 0$ and $x_L^\sigma = -\frac{\alpha_\sigma}{1-\alpha_\sigma}\frac{c}{r}\frac{1}{\nu}$. A direct computation shows that the mapping $\sigma \longrightarrow x\nu - \frac{c}{r} + \left(\frac{c}{r} - x_L^\sigma \nu\right)\left(\frac{x}{x_L^\sigma}\right)^{\alpha_\sigma}$ is increasing on $(0, \infty)$. Lemma 2 is then deduced remarking that, if $\mu_A = \mu_B$, then $x_L^A > x_L^B$ and thus $E_B(x) > E_A(x) = 0 \;\; \forall x_L^B < x < x_L^A$. □

Proof of Lemma 3 A sufficient condition for obtaining our result is $E'_A(x) > E'_B(x)$ for all $x > x_L^B$. We have for all $x > x_L^B$:

$$
\begin{aligned}
\frac{1}{1-\theta}\left(xE'_A(x) - xE'_B(x)\right) &= x(\nu_A - \nu_B) + \alpha_A\left(\frac{c}{r} - x_L^A\nu_A\right)\left(\frac{x}{x_L^A}\right)^{\alpha_A} \\
&\quad -\alpha_B\left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B} \\
&> x_L^B(\nu_A - \nu_B) + \alpha_A\left(\frac{c}{r} - x_L^A\nu_A\right)\left(\frac{x}{x_L^B}\right)^{\alpha_A} \\
&\quad -\alpha_B\left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B} \\
&> \left\{x_L^B(\nu_A - \nu_B) + \alpha_A\left(\frac{c}{r} - x_L^A\nu_A\right)\right. \\
&\quad \left. -\alpha_B\left(\frac{c}{r} - x_L^B\nu_B\right)\right\}\left(\frac{x}{x_L^B}\right)^{\alpha_A} \\
&> \left\{\left(\nu_A x_L^A - \frac{c}{r}\right)(1-\alpha_A) - \left(\nu_B x_L^B - \frac{c}{r}\right)(1-\alpha_B)\right\} \\
&\quad \times\left(\frac{x}{x_L^B}\right)^{\alpha_A} = 0
\end{aligned}
$$

□

Proof of Proposition 1 It follows from the strong Markov property that optimization problem (11) can be rewritten under the form

$$
E(x) \equiv \sup_{\tau_S \in \mathcal{T}} \mathbb{E}\left[\int_0^{\tau_S} \mathrm{e}^{-rt}(1-\theta)\left(X_{t,A}^x - c\right)\mathrm{d}t \; + \; \mathrm{e}^{-r\tau_S}E_B(X_{\tau_S,A}^x)\right].
$$

The proof [6] of our proposition relies then on the following lemma which shows that the optimal switching strategy is a trigger strategy. □

Lemma 4 *If*

$$E(x) = \mathbb{E}\left[\int_0^{\tau_L^B} \mathrm{e}^{-rt}(1-\theta)\left(X_{t,B}^x - c\right)\mathrm{d}t\right]$$

then,

$$E(x-h) = \sup_{\tau_S\in\mathcal{T}} \mathbb{E}\left[\int_0^{\tau_S} \mathrm{e}^{-rt}(1-\theta)\left(X_{t,A}^{x-h} - c\right)\mathrm{d}t + \mathrm{e}^{-r\tau_S}E_B(X_{\tau_S,A}^{x-h})\right]$$
$$= \mathbb{E}\left[\int_0^{\tau_L^B} \mathrm{e}^{-rt}(1-\theta)\left(X_{t,B}^{x-h} - c\right)\mathrm{d}t\right]$$

Proof of the Lemma 4 Taking advantage from the equalities $X_{t,A}^{x-h} = X_{t,A}^x - X_{t,A}^h$ and $X_{t,B}^{X_{\tau,A}^{x-h}} = X_{t,B}^{X_{\tau,A}^x} - X_{t,B}^{X_{\tau,A}^h}$, we deduce from the definitions of $E(x)$ and $E(x-h)$

$$E(x-h) \le E(x) - \inf_{\tau\in\mathcal{T}}\left\{(1-\theta)\mathbb{E}\left[\int_0^{\tau} \mathrm{e}^{-rt}X_{t,A}^h\mathrm{d}t\right.\right.$$
$$\left.\left.+\mathrm{e}^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B}\mathrm{e}^{-rt}X_{t,B}^{X_{\tau,A}^h}\mathrm{d}t|\mathcal{F}_\tau\right]\right]\right\}$$

Moreover,

$$\mathbb{E}\left[\int_0^{\tau}\mathrm{e}^{-rt}X_{t,A}^h\mathrm{d}t\right] = \nu_A\left(h - \mathbb{E}\left[\mathrm{e}^{-r\tau}X_{\tau,A}^h\right]\right),$$
$$\mathbb{E}\left[\int_0^{\tau_L^B}\mathrm{e}^{-rt}X_{t,B}^{X_{\tau,A}^h}\mathrm{d}t \mid \mathcal{F}_\tau\right] = \nu_B\left(X_{\tau,A}^h - x_L^B\mathbb{E}\left[\mathrm{e}^{-r\tau_L^B}\mid\mathcal{F}_\tau\right]\right),$$

from which we deduce

$$\mathbb{E}\left[\int_0^{\tau}\mathrm{e}^{-rt}X_{t,A}^h\mathrm{d}t + \mathrm{e}^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B}\mathrm{e}^{-rt}X_{t,B}^{X_{\tau,A}^h}\mathrm{d}t\mid\mathcal{F}_\tau\right]\right]$$
$$= \nu_A h - (\nu_A-\nu_B)\,\mathbb{E}\left[\mathrm{e}^{-r\tau}X_{\tau,A}^h\right] - \nu_B x_L^B\mathbb{E}\left[\mathrm{e}^{-r\tau}\mathrm{e}^{-r\tau_L^B}\right]$$

[6] Our problem is actually a particular case of a more general (and standard) problem in optimal stopping theory which is stated and solved in Theorem 10.4.1, Oksendal (2003). We propose here an elementary proof of our (simple) problem.

Now, from a standard result in optimal stopping theory we have that, $\sup_{\tau\in\mathcal{T}} \mathbb{E}\left[\mathrm{e}^{-r\tau} X^h_{\tau,A}\right] = h$ which implies that

$$\inf_{\tau\in\mathcal{T}} \left\{\mathbb{E}\left[\int_0^{\tau} \mathrm{e}^{-rt} X^h_{t,A}\mathrm{d}t + \mathrm{e}^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B} \mathrm{e}^{-rt} X^{X^h_{\tau,A}}_{t,B}\mathrm{d}t|\mathcal{F}_\tau\right]\right]\right\}$$
$$= \mathbb{E}\left[\int_0^{\tau_L^B} \mathrm{e}^{-rt} X^h_{t,B}\mathrm{d}t\right].$$

We thus obtain

$$E(x-h) \le \mathbb{E}\left[\int_0^{\tau_L^B} \mathrm{e}^{-rt}(1-\theta)\left(X^{x-h}_{t,B} - c\right)\mathrm{d}t\right].$$

As the converse inequality is always satisfied, lemma(4) is proved. □

Thus, the optimal switching policy is a trigger policy. For a given switching trigger x_S, the equity value is given by standard computations

$$\begin{cases} E(x) = (1-\theta)\Big\{x\nu_A - \frac{c}{r} - x_S(\nu_A - \nu_B)\left(\frac{x}{x_S}\right)^{\alpha_A} \\ \qquad + \left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_A}\left(\frac{x_S}{x_L^B}\right)^{\alpha_B}\Big\} & \text{if } x > x_S, \\ E(x) = (1-\theta)\left\{x\nu_B - \frac{c}{r} + \left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B}\right\} & \text{if } x_L^B < x \le x_S, \\ E(x) = 0 & \text{if } x < x_L^B \end{cases}$$

It is easy to see that this value function reaches its maximum for a value of x_S that does not depend on x, namely

$$x_S^\star = \left(\frac{(\alpha_B - \alpha_A)\nu_B}{(\nu_A - \nu_B)(1-\alpha_A)(-\alpha_B)}\right)^{\frac{1}{1-\alpha_B}} x_L^B > x_L^B.$$

Proof of Proposition 2 Since by construction $E_A\left(x_L^{NS}\right) = E_B\left(x_L^{NS}\right) = 0$, a necessary condition for equityholders being not tempted by switching is $E'_A(x_L) > E'_B(x_L)$ where x_L is a liquidation trigger. The minimum liquidation trigger that satisfies this condition is implicitly defined by the equation $x_L E'_A(x_L) = x_L E'_B(x_L)$. This leads to $x_L = x_L^{NS} = \frac{c}{r}\frac{\alpha_B - \alpha_A}{\nu_A(1-\alpha_A) - \nu_B(1-\alpha_B)}$. Conversely, reasoning as in the proof of Lemma 3, we show that $E_A(x) - E_B(x) \ge 0$ for $x \ge x_L^{NS}$. □

Acknowledgements We thank Catherine Casamatta and two anonymous referees for thoughtful comments and suggestions. We would like also to thank participants at the 9th International Conference on Real options, at the EEA meeting, Amsterdam 2005 and at the ASSET meeting, Crete 2005. Financial supports from FNS and Europlace Institute of Finance are gratefully acknowledged by the authors.

References

Anderson, R., Sundaresan, S.: Design and valuation of debt contracts. Rev. Financ. Stud. **9**, 37–68 (1996)
Bliss, R.: Market discipline and subordinated debt: a review of some salient issues, Federal Reserve Bank of Chicago. Econ. Perspect. **25**, 24–45 (2001)
Childs, P.D., Mauer, D.C., Ott, S.H.: Interactions of corporate financing and investment decisions: the effects of agency conflicts. J. Financ. Econ. **76**, 667–690 (2005)
Décamps, J.P., Faure-Grimaud, A.: Excessive continuation and dynamic agency costs of debt. Eur. Econ. Rev. **46**, 1623–1644 (2002)
Ericsson, J.: Asset substitution, debt pricing, optimal leverage and maturity. Finance **21**, 39–70 (2000)
Fan, H., Sundaresan, S.: Debt valuation, renegotiation, and optimal dividend policy. Rev. Financ. Stud. **13**, 1057–1099 (2000)
Goldstein, R., Ju, N., Leland, H.: An EBIT based dodel of dynamic capital structure. J. Bus. **74**, 483–512 (2001)
Graham, J.: How big are the tax benefits of debt? J. Finance **55**, 1901–1941 (2000)
Henessy, C.A., Tserlukevich, Y.: Dynamic hedging incentives, debt and warrants. Haas School of Business, Berkeley (2004)
Jensen, M., Meckling, W.: Theory of the firm: managerial behavior, agency costs and ownership structure. J. Financ. Econ. **3**, 305–360 (1976)
Ju, N., Ou-Yang, H.: Asset substitution and underinvestment: a dynamic view. Fuqua School of Business, Duke University (2005)
Kim, I.J., Ramaswamy, K., Sundaresan, S.: Does default risk in coupons affect the valuation of corporate bonds?: a contingent claims model. Financ. Manage. **22**, 117–131 (1993)
Leland, H.: Corporate debt value, bond covenants, and optimal capital structure. J. Finance **49**, 1213–1252 (1994)
Leland, H.: Agency costs, risk management, and capital structure. J. Finance **53**, 1213–1243 (1998)
Leland, H., Skarabot, J.: On purely financial synergies and the optimal scope of the firm: implications for mergers, spin-offs, and structured finance, Haas School of Business. University of California, Berkeley (2004)
Mauer, D.C., Sarkar, S.: Real options, agency conflicts, and optimal capital structure. J. Bank. Finance **29**, 1405–1428 (2005)
Mauer, D.C., Triantis, A.J.: Interactions of corporate financing and investment decisions: a dynamic framework. J. Finance **49**, 1253–1277 (1994)
Mello, A.S., Parsons, J.E.: Measuring the agency costs of debt. J. Finance **47**, 1887–1904 (1992)
Oksendal, B.: Stochastic differential equations. Springer, Berlin Heidelberg New York (2003)
Parrino, R., Weisbach M.S.: Measuring Investment distorsion arising from stockholder–bondholder conflicts. J. Financ. Econ. **53**, 3–42 (1999)